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POWER VS. LOGARITHMIC MODEL OF FITTS' LAW: A MATHEMATICAL ANALYSIS[1]

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RÉSUMÉ – Modèle de puissance vs. logarithmique de la loi de Fitts : une analyse mathématique. *Après bientôt soixante années d'études, il reste toujours à déterminer si la loi de Fitts, un modèle célèbre du mouvement de pointage humain, est une loi logarithmique ou de puissance. Dans deux articles abondamment cités, Meyer & al. ont avancé l'idée que le modèle de puissance qu'ils ont déduit de leur théorie stochastique des sous-mouvements optimisés englobe le modèle logarithmique comme un cas limite atteint lorsque le nombre de sous-mouvements devient grand. Reconsidérant la théorie des sous-mouvements de Meyer et al., nous montrons que cette proposition est mathématiquement inexacte. La théorie de Meyer et al. implique en réalité un modèle quasi-logarithmique plutôt que de puissance, le premier n'étant pas équivalent au second. Une pleine conscience que les deux classes possibles de description mathématique de la loi de Fitts ne sont pas équivalentes nous semble de nature à stimuler la recherche expérimentale dans ce domaine.*

MOTS-CLÉS – Loi de Fitts, Équations fonctionnelles, Modèles mathématiques en psychologie, Pointage, Mouvement rapide orienté vers une cible

SUMMARY – *Whether Fitts' law, a well-known model of human pointing movement, is a logarithmic law or a power law has remained unclear so far. In two widely cited papers, Meyer & al. have claimed that the power model they derived from their celebrated stochastic optimized-submovement theory encompasses the logarithmic model as a limiting case, when the number of submovements grows large. We review the Meyer & al. submovement theory and show that this claim is questionable mathematically. Our analysis reveals that Meyer & al.'s theory implies in fact a quasi-logarithmic, rather than quasi-power model, the two models not being equivalent. Awareness that the two classes of candidate mathematical descriptions of Fitts' law are not equivalent should stimulate experimental research in the field.*

KEYWORDS – Fitts' law, Functional equations, Mathematical models in psychology, Pointing, Simple rapid aimed movement

1. INTRODUCTION

Fitts' law [Fitts, 1954; Fitts & Peterson, 1964; Plamondon & Alimi, 1997] is a well-known empirical regularity which predicts the average movement time T it takes people, under time pressure, to reach a target of width W located at distance D

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(see Figure 1) with some pointer. This model has proven useful in several fields of applied psychology such as Human-Computer Interaction and Ergonomics.



FIGURE 1. One-dimensional single-shot movement paradigm settled by Fitts [Fitts & Peterson, 1964]. Starting at origin o, the pointer must be moved rapidly to target center c, the target being displayed as a tolerance interval of width W.

The mathematical formulation of Fitts' law is as follows: movement time T is linearly dependent on an *index of difficulty* [Fitts & Peterson, 1964] ID, which in turn is a strictly increasing function f of the relative distance D/W. Thus

$$T = a + b \cdot f(D/W),$$

where a and b are adjustable constants. The question is to know what the shape of f is. Many different formulations have been proposed for f in the literature over the past fifty years, none of which has been empirically disproved. Perhaps the most favorably received are:

$$\mathrm{ID} = f(D/W) = \begin{cases} \log_2 \dfrac{2D}{W} & \text{Fitts [1954]} \quad (1a) \\ \log_2\bigl(0.5 + \dfrac{D}{W}\bigr) & \text{Welford [1960]} \quad (1b) \\ \log_2\bigl(1 + \dfrac{D}{W}\bigr) & \text{McKenzie [1989]} \quad (1c) \\ \sqrt{\dfrac{D}{W}} & \text{Meyer } \textit{et al.} \text{ [1988]} \quad (1d) \\ \Bigl(\dfrac{D}{W}\Bigr)^{1/n} & \text{Meyer } \textit{et al.} \text{ [1990]}^3 \quad (1e) \end{cases}$$

In particular, whether Fitts' law is a logarithmic (1a)–(1c) or a power law (1d)–(1e) has remained unclear so far. One plausible explanation is that the practical range of relative distances D/W that can be actually investigated in the laboratory is rather narrow [Guiard & Olafsdottir, 2011]:

$$3 \lesssim \frac{D}{W} \lesssim 33. \tag{2}$$

This is because for $D/W \ll 3$ experimenters have a floor effect on T (speed saturation) while for $D/W \gg 33$ the error rate tends to explode (accuracy saturation). Within the range $[3, 33]$, the curves (1a)–(1e) are indeed similar (see Figure 2), making it difficult to decide empirically between the logarithmic and the power model.

[3]Here n is an integer representing the maximum "number of submovements" [Meyer *et al.*, 1990] and (1d) corresponds to $n = 2$. The paternity of the power law was later claimed by Kvålseth [1993], who showed that the formulation $T = a(D/W)^b$ achieves higher fits to Fitts' data [Kvålseth, 1980].

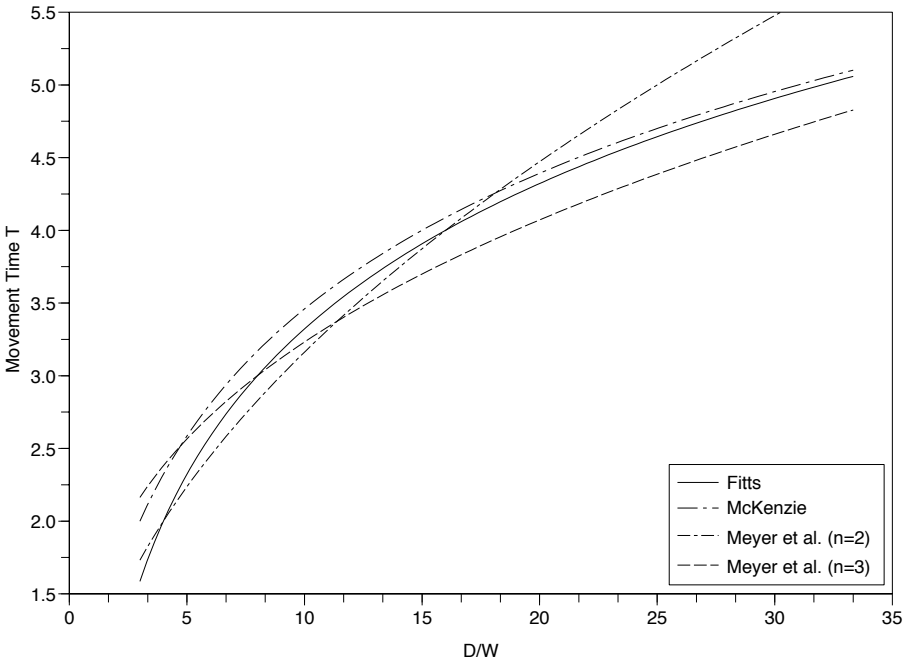


FIGURE 2. Movement time T vs. D/W for the four formulations of (1) in the range (2). Eq. (1d) yields a square-root law while Eq. (1e) is plotted with $n = 3$ (cube-root law).

Such irresolution would not be a problem if mathematical models were just black boxes serving to deliver numerical simulations of empirical data – that minimal job is done about equally well by the logarithmic and the power model of Fitts' law. Thus, from a practical perspective, one could always support the claim that one mathematical model or another accurately approximates the data.

But our concern is more theoretical. In general Fitts' law researchers do care about the internal structure of their mathematical formulas, which they typically take as models of the law they want to understand. Ideally a mathematical model, as distinct from a blind numerical simulation, is a short statement whose variables, parameters, and relations explicitly map onto quantities and relations of some substantive theory. For example, in (1c) the term $\log_2(1 + D/W)$ is meant to represent a measure of Shannon's information capacity [Shannon, 1948] and so the logarithmic form of that model is a deliberate, theoretically-driven option. Likewise, it is an elaborate theoretical analysis that led Meyer *et al.* [1990] to put forth the power equation of (1e), in which the parameter n is supposed to represent the maximum number of submovements a participant is willing to make.

Despite the similarity of predictions within the range of interest (2), the logarithmic vs. power form of Fitts' law is indeed a theoretically-important empirical issue, and so there is reason to be concerned by the lack of consensus. To paraphrase Platt [1964, p. 351], "*a failure to agree for 60 years is public advertisement of a failure to disprove.*"

In two widely cited papers Meyer *et al.* [Meyer *et al.*, 1988, 1990] argued that $a+b\cdot(D/W)^{1/n}$ tends to $a'+b'\cdot\log(D/W)$ as the maximum number n of submovements tends to infinity. This suggests that there is no real logarithmic vs. power issue

about Fitts' law: the power model they derived from their celebrated stochastic optimized-submovement theory would encompass the logarithmic model as a limiting case.

It turns out, however, that this claim is questionable. We propose a detailed analysis to show that Meyer *et al.*'s model does not predict a genuine power law but rather some quasi-logarithmic law, the two classes of candidate mathematical descriptions of Fitts' law being not equivalent.

This paper is organized as follows. In Section 2, after reviewing some mathematical derivations of Fitts' law based on submovement models, we restate Meyer *et al.*'s claim and disprove it. We then review the stochastic optimized-submovement model in Section 3 to explain analytically why the power formulation fails as a mathematical description of Fitts' law for multiple submovements. Section 4 concludes.

2. PREVIOUS WORK AND A DISPROOF

We begin with a short review of two major mathematical derivations of Fitts' law. Both assume that the pointing movement toward the target is composed of a sequence of n submovements.

2.1. DETERMINISTIC ITERATIVE-CORRECTIONS MODEL

The deterministic iterative-corrections model was originally formulated by Crossman and Goodeve [1963]. The model assumes that each submovement (i) requires a constant time ΔT to be executed and (ii) covers a constant fraction $\lambda < 1$ of the remaining distance to the center of the target. The target is reached if that remaining distance is equal to half the target width $W/2$.

It follows that the required time $T = n\ \Delta T$ to reach the target is such that $\lambda^n D = W/2$. This is easily seen to imply

$$T = c \log_2 \frac{2D}{W}$$

where $c = \Delta T \log_2(1/\lambda)$, that is, a logarithmic formulation of the form (1a).

This model, while being very simple, has long been abandoned because it relies on rather strong assumptions (i,ii) that have been empirically observed to be wrong [Plamondon & Alimi, 1997, § 2.1.2]. Also, one fatal objection is that the model ignores submovement endpoint variability, and thus fails to predict any movement endpoint spread and to accommodate the possibility of target misses, most notably overshoots [Meyer *et al.*, 1988, 1990; Plamondon & Alimi, 1997].

2.2. STOCHASTIC OPTIMIZED-SUBMOVEMENT MODEL

The stochastic optimized-submovement model proposed by Meyer *et al.* [1988] assumes a random spread of submovement endpoints. The authors follow Woodworth's 1899 suggestion [Woodworth, 1899] that the movement involves $n = 2$ successive phases[4]: There is an initial 'ballistic' submovement for which the endpoint spread is

[4]The model was also extended to multiple submovements (arbitrary large n): see Section 3.

proportional to velocity, and whose duration is given by [Smith, 1988]

$$T_i = \frac{D/W - 1/2}{s}$$

where s is some spread parameter[5]. Let Δ be the resulting distance to the target center; Δ is modeled by a *stochastic* (random) variable – following, e.g., a Gaussian distribution – whose standard deviation is proportional to s.

Next, if the target is not reached yet ($|\Delta| > W/2$), a secondary submovement occurs. The total average time (within a constant multiplicative factor) is the sum:

$$T = \min_s \Big\{ \frac{D/W - 1/2}{s} + \mathbf{E}_{|\Delta|>W/2}\Big(\frac{|\Delta|}{W}\Big) \Big\} \tag{3}$$

where the expectation $\mathbf{E}$ is with respect to Δ's distribution over the region $|\Delta| > W/2$. The optimization takes the form of a minimization over s, the only free variable left [Meyer *et al.*, 1988, appendix].

Meyer *et al.* found that the solution is proportional to

$$T \propto \frac{2\theta\sqrt{D/W} - \sqrt{W/D}}{\theta\sqrt{\theta - W/D}} \tag{4}$$

where θ is in fact a complicated function of D/W such that $\theta = k \exp \frac{1}{2(\theta D/W - 1)}$ and is determined iteratively by a fixed point theorem [Meyer *et al.*, 1988, appendix] for a given value of proportionality factor k. The result is well aproximated by a square-root law of the form (1d) (see Figure 3).

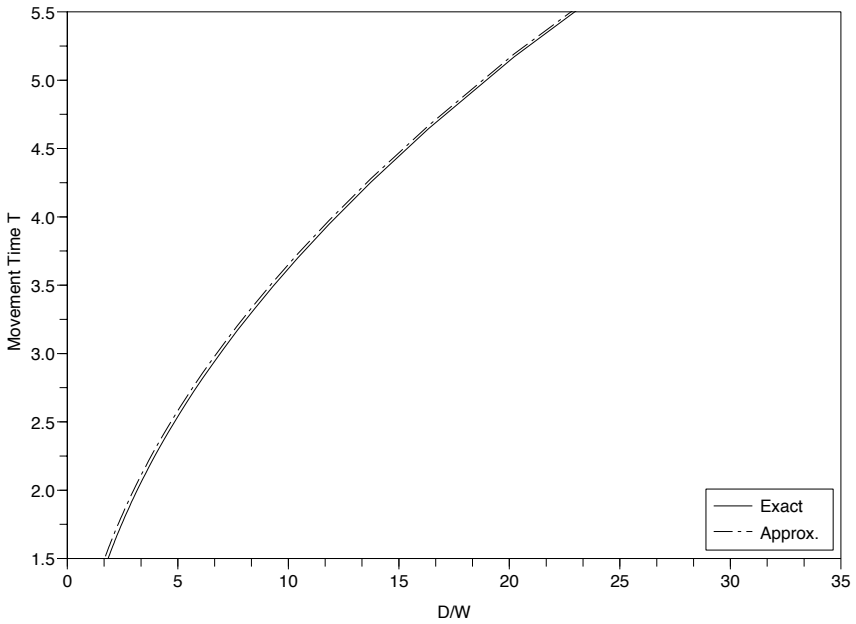


FIGURE 3. Movement time T vs. D/W for the exact solution (4) (solid) and corresponding square-root approximation (dashed).

[5]Following [Smith, 1988], a term $1/2$ is introduced so that $T = 0$ when the origin already lies in the target.

The stochastic optimized-submovement model can be easily extended to multiple submovements (see subsection 3.1 below). Meyer *et al.* [1990] claimed that for n submovements, the model yields a solution that can likewise be approximated by a nth root law of the form (1e).

2.3. Meyer *et al.*'s [1988, 1990] claim for multiple submovements: a disproof

The claim is as follows (see Figure 4):

> *Mathematically,* $\log_2(D/W)$ *is equivalent to the limiting case of a power function* $(D/W)^x$ *of* D/W *whose positive exponent* x *tends to zero* [Meyer *et al.*, 1988, footnote 13].

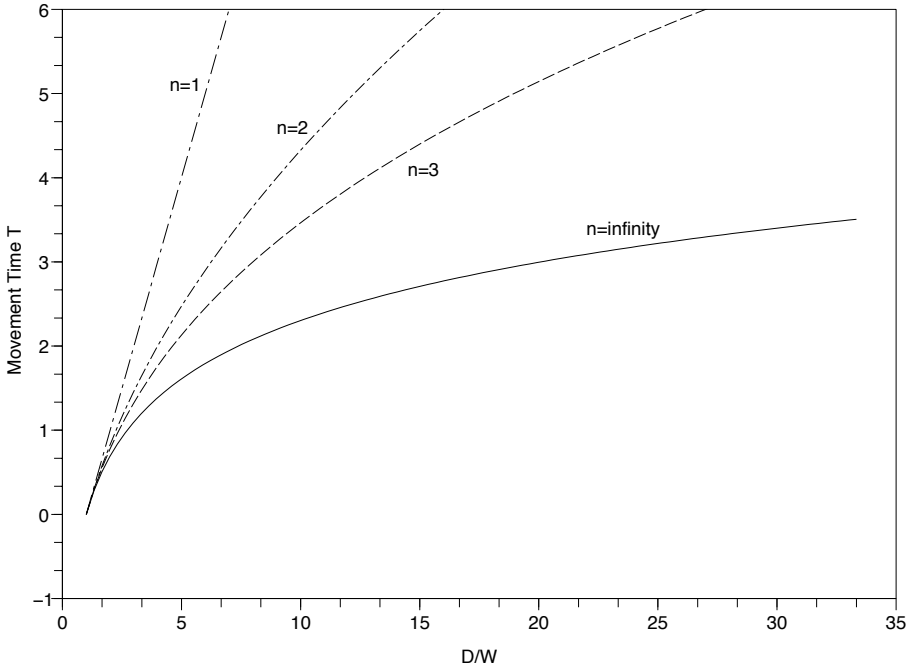


FIGURE 4. A remake of Meyer *et al.*'s [1990] original Figure 6.13 giving their hypothetical T vs. D/W. They claim that as n grows large, their power relation "*approaches a logarithmic function, paralleling Fitts' Law*" [Meyer *et al.*, 1990].

Meyer *et al.*'s widely cited papers [1988, 1990][6] have convinced the Fitts' law research community that their power law encompasses the logarithmic law at the extreme case $n = \infty$ (see e.g. Wobbrock *et al.* [2008, Eq. (18)] for a recent account).

It turns out, however, that the above claim is mathematically questionable. For any fixed value of $D/W > 0$, when the exponent $x = 1/n$ tends to zero,

$$\sqrt[n]{D/W} = (D/W)^{1/n} = \exp\Big(\frac{1}{n}\log_e(D/W)\Big)$$

[6]At the time of this writing, [Meyer *et al.*, 1988, 1990] received 744 and 225 citations, respectively. Source: Google Scholar.

has first order approximation $1 + \frac{1}{n}\log_e(D/W)$ but tends to $\exp 0 = 1$ as $n \to +\infty$. Hence the limit is a 'constant law', which is not even a strictly increasing function of D/W and is, therefore, inadequate as a model for Fitts' law[7]. This was apparently first noticed by one of us [Guiard *et al.*, 2001, footnote 6].

Since we took (1e) for granted, our disproof raises the question of the actual validity of the power law. Also, since a square-root or cube-root function is certainly not constant, it makes the dependence on n questionable: different nth root laws (1e) appear different for different values of n, including large ones. In the next section, we attempt to solve these problems.

3. DETAILED ANALYSIS FOR n SUBMOVEMENTS

We now review the submovement theory of Meyer *et al.* for multiple submovements [Meyer *et al.*, 1990; Smith, 1988] to explain analytically why the nth-root model fails.

3.1. DERIVATION

Let $T = f_n(D/W)$ be the average movement time required to reach the target after n submovements. After the initial 'ballistic' submovement, there remain $n-1$ submovements to reach the target located at random distance $|\Delta|$. The stochastic optimized-submovement model (3) then predicts:

$$f_n\Big(\frac{D}{W}\Big) = \min_s\Big\{\frac{D/W - 1/2}{s} + \mathbf{E}_{|\Delta|>W/2} f_{n-1}\Big(\frac{|\Delta|}{W}\Big)\Big\} \tag{5}$$

for any $n > 1$. To simplify the notation let δ denote any value of D/W and let t denote any time value. To simplify the calculations we follow Smith's [1988] assumption that Δ's distribution is uniform in the interval $(-Ws/2, Ws/2)$[8]:

$$f_n(\delta) = \min_s\Big\{\frac{\delta - 1/2}{s} + \frac{2}{s}\int_{1/2}^{s/2} f_{n-1}(\delta)\mathrm{d}\delta\Big\} \tag{6}$$

Note that the factor 2 accounts for undershoots as well as overshoots. It is easily seen by induction that f_n is well defined, regular (indefinitely continously differentiable) and strictly increasing in the range $\delta > 1/2$. We can, therefore, define its inverse function $\delta = g_n(t)$. It turns out [Smith, 1988] that the determination of relative distance vs. time (that is, of g_n) is easier than the direct determination of f_n, that is, of time vs. relative distance as in the classical formulation of Fitts' law.

[7]The illustration made in [Meyer *et al.*, 1990, Figure 6.13] (see Figure 4) would suggest a weaker claim: there exist sequences $(a_n), (b_n)$ of real numbers with $b_n > 0$ and constants a and $b > 0$ such that $\lim_{n\to\infty}\big(a_n + b_n\big(\frac{D}{W}\big)^{1/n}\big) = a + b\cdot\log_e\big(\frac{D}{W}\big)$. This is in fact possible by setting $b_n = a - a_n = n\cdot b$ for all n, so that the curves all intersect at $D/W = 1$. During the review process, we became aware that a similar statement was put forth by Kvålseth [1993]. This implies, however, that $(a_n), (b_n)$ are unbounded sequences so that $T = a_n + b_n\big(\frac{D}{W}\big)^{1/n}$ is not a genuine power model: Section 3 offers an explanation.

[8]Calculations run similarly for other distributions (e.g., Gaussian), with just more intricate results.

We now derive a simple proof leading to Smith's solution to (6). Making the first derivative of (6) vanish, the optimal $s = s(\delta)$ satisfies

$$-\frac{\delta - 1/2}{s^2} - \frac{2}{s^2}\int_{1/2}^{s/2} f_{n-1}(\delta)\,\mathrm{d}\delta + \frac{1}{s} f_{n-1}(\tfrac{s}{2}) = 0,$$

which boils down to the condition

$$f_n(\delta) = f_{n-1}(s/2). \tag{7}$$

Now, by inverting arguments according to $\delta = g_n(t)$, (6) can be rewritten as

$$t = \frac{g_n(t) - 1/2}{s} + \frac{2}{s}\int_{g_{n-1}(0)}^{g_{n-1}(t)} f_{n-1}(\delta)\mathrm{d}\delta \tag{8}$$

$$t = \frac{g_n(t) - 1/2}{s} + \frac{2}{s}\big(\frac{s}{2}\cdot t - \int_0^t g_{n-1}(\tau)\mathrm{d}\tau\big) \tag{9}$$

where we have used (7) in the form $g_{n-1}(t) = s/2$ in (8) and the inverse function integration theorem (see Figure 5) in (9). After subtracting t on both sides of (9)

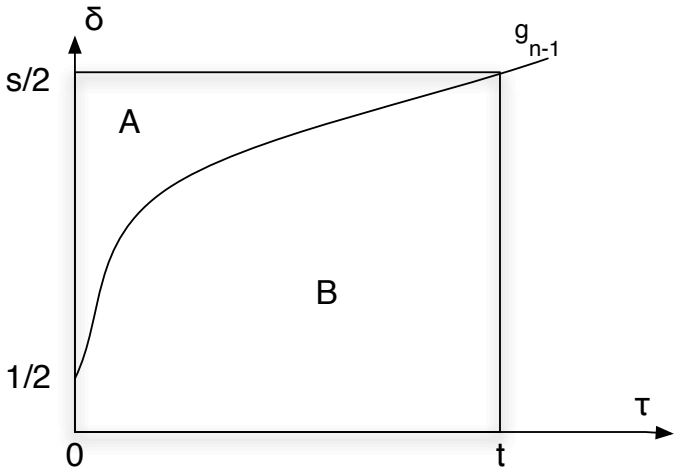


FIGURE 5. The inverse function integration theorem states that the sum of the two areas $A = \int_{1/2}^{s/2} f_{n-1}(\delta)\mathrm{d}\delta$ and $B = \int_0^t g_{n-1}(\tau)\mathrm{d}\tau$ is equal to $A + B = \frac{s}{2}\cdot t$, the area of the rectangle.

we end up with a simple recursion relation

$$g_n(t) = \frac{1}{2} + 2\int_0^t g_{n-1}(\tau)\mathrm{d}\tau$$

which is easily solved by induction. One finds:

$$g_n(t) = \frac{1}{2} + \frac{1}{2}(2t) + \frac{1}{2}\frac{(2t)^2}{2} + \cdots + \frac{1}{2}\frac{(2t)^n}{n!} = \frac{E_n(2t)}{2}, \tag{10}$$

where

$$E_n(t) = 1 + t + \frac{t^2}{2!} + \cdots + \frac{t^n}{n!} \tag{11}$$

denotes the nth partial sum of the Taylor series of the exponential $\exp(t)$. Denoting its inverse function by $L_n = E_n^{-1}$, one obtains the following law:

$$\boxed{T = \frac{1}{2} L_n\big(2\frac{D}{W}\big).} \tag{12}$$

3.2. CLOSED-FORM EXPRESSIONS

For each n, the exact solution (12) is obtained by solving an nth degree equation of the form $E_n(t) = 0$. We can compute L_n for the first few values of n.

For $n = 2$ one immediately obtains

$$L_2(x) = \sqrt{2x - 1} - 1$$

which implies

$$T = \sqrt{D/W - 1/4} - 1/2.$$

Thus one recovers the quasi square-root law of Meyer *et al.* [1988].

For $n = 3$ one obtains

$$L_3(x) = \sqrt[3]{3x + \sqrt{9x^2 - 6x + 2} - 1} - \frac{1}{\sqrt[3]{3x + \sqrt{9x^2 - 6x + 2} - 1}} - 1,$$

which is a rather complex "quasi cube-root law:"

$$T = \frac{1}{2}\sqrt[3]{6\tfrac{D}{W} + \sqrt{36(\tfrac{D}{W})^2 - 12\tfrac{D}{W} + 2} - 1} - \frac{1}{2\sqrt[3]{6\tfrac{D}{W} + \sqrt{36(\tfrac{D}{W})^2 - 12\tfrac{D}{W} + 2} - 1}} - \frac{1}{2}.$$

The case $n = 4$:

$$L_4(x) = \frac{1}{2\sqrt{\dfrac{16(2x-1)}{\sqrt[3]{192x+32\sqrt{32x^3-12x^2+12x-3}-32}} - \sqrt[3]{192x+32\sqrt{32x^3-12x^2+12x-3}-32} - \dfrac{16}{\sqrt{-\dfrac{32x-\sqrt[3]{(192x+32\sqrt{32x^3-12x^2+12x-3}-32)^2}+4\sqrt[3]{192x+32\sqrt{32x^3-12x^2+12x-3}-32}-16}{\sqrt[3]{192x+32\sqrt{32x^3-12x^2+12x-3}-32}}}} - 8} + \frac{1}{2\sqrt{-\dfrac{32x-\sqrt[3]{(192x+32\sqrt{32x^3-12x^2+12x-3}-32)^2}+4\sqrt[3]{192x+32\sqrt{32x^3-12x^2+12x-3}-32}-16}{\sqrt[3]{192x+32\sqrt{32x^3-12x^2+12x-3}-32}}}} - 1$$

cannot be easily identified with a "fourth-root law" and is probably too intricate to be of any practical use.

For $n > 4$ the equation $E_n(t) = 0$ is not even solvable by radicals and so there exists no closed-form expression. It follows that for large n, (12) *cannot* be identified with an nth-root (power) formulation of Fitts' law. Thus, the conclusion of Meyer *et al.* [1988, 1990] that Fitts' logarithmic formulation arises out of a power law with an infinite number of submovements is doubtful. The next section confirms that doubt.

3.3. QUASI-POWER VS. QUASI-EXPONENTIAL LAWS

In our derivation, the case $|\Delta| \leq W/2$ (target is reached) implies a total number of submovements strictly less than n. Therefore, n appears as the *maximum* number of permitted submovements and it would be desirable to let $n \to +\infty$ to obtain a general formulation of Fitts' law accounting for any number of submovements.

Similarly to the case $n = 2$ [Meyer, 1988], one could argue that when D/W is large (hence T is large) (11) can be approximated by its highest-degree term:

$$\frac{D}{W} = \frac{1}{2}E_n(2T) \approx \frac{1}{2}\frac{(2T)^n}{n!} \tag{13}$$

so that (12) is indeed approximated by an *nth root law*:

$$T = \frac{1}{2}\sqrt[n]{n!\frac{D}{W}}. \tag{14}$$

However, this is not a genuine power model since as $n \to +\infty$, the multiplying slope factor explodes: $\sqrt[n]{n!} \sim n/e \to +\infty$ (see Figure 6).

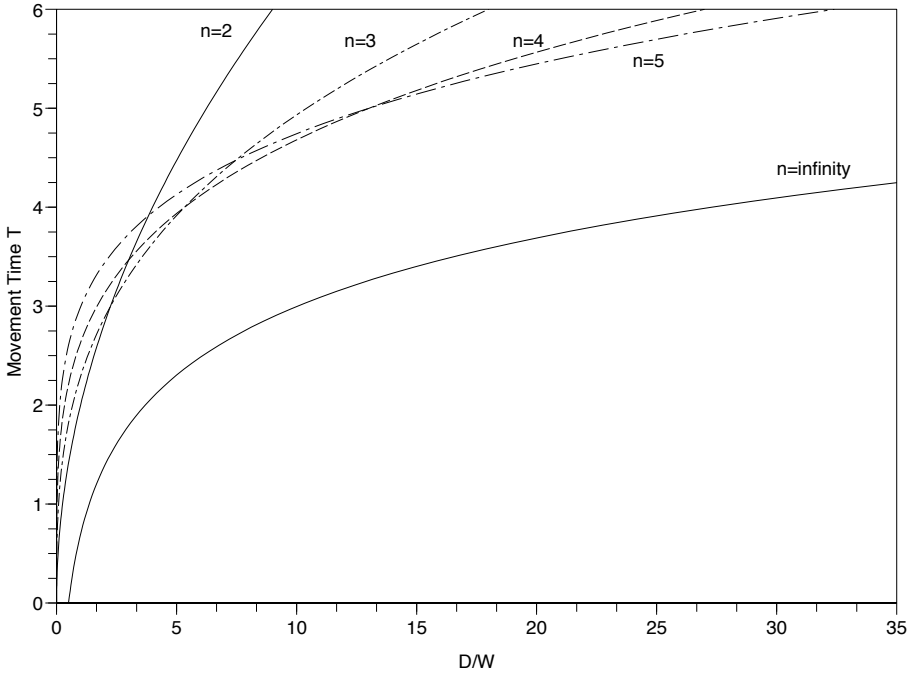


FIGURE 6. Approximated nth-root laws (14) for increasing values of n.

In contrast, as Smith [1988] noticed, for any value of D/W (including small ones), the partial sum (11) rapidly converges to the exponential as $n \to +\infty$:

$$\frac{D}{W} = \frac{1}{2}E_n(2T) \longrightarrow \frac{1}{2}\exp(2T), \tag{15}$$

and so the final result is logarithmic (see Figure 7):

$$T = \frac{1}{2}\log_e\left(2\frac{D}{W}\right). \tag{16}$$

Comparison of Figures 6 and 7 shows that even for particular moderate values of n, the nth-root power function does not approximate the stochastic optimized-submovement model's predicted movement times. The exact solution (12) is rather quasi-logarithmic and rapidly converges to the logarithmic law (16).

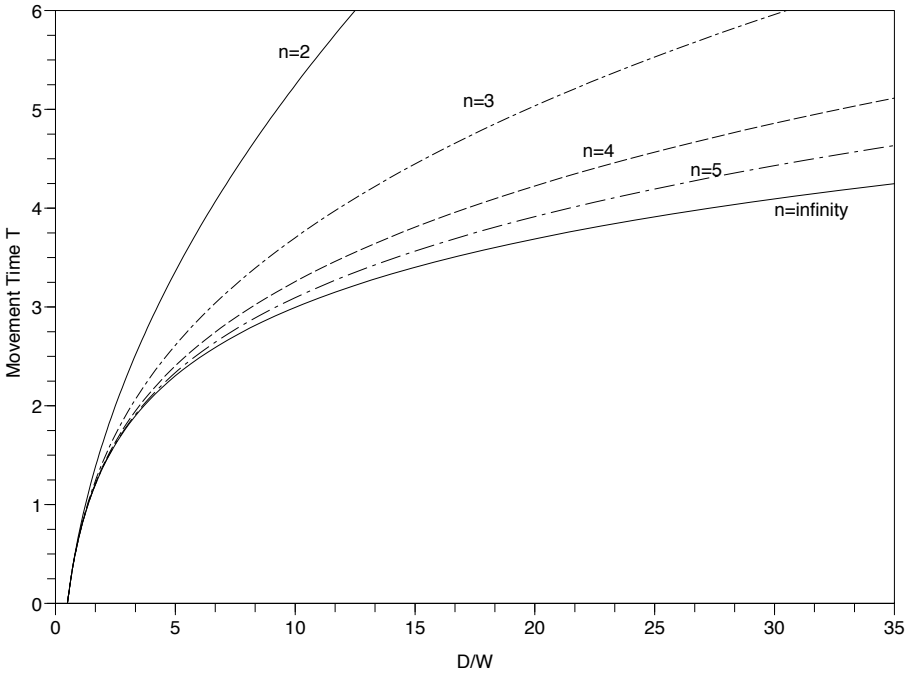


FIGURE 7. Exact laws (12) rapidly converging to the logarithmic law (16) as $n \to \infty$.

4. CONCLUSION

Our mathematical analysis shows that for multiple submovements, the solution of the stochastic optimized-submovement model cannot be identified with a power law of the form (1e), even to a rough approximation, as the number n of submovements grows large. Not only does the power model fail to encompass the logarithmic model, the submovement theory does not yield a genuine power law; it is, rather, a quasi-logarithmic law (12), which rapidly converges to a logarithmic law of the form (1a) or (16).

Since Meyer *et al.*'s stochastic optimized submovement theory [1988, 1990] is generally considered the best explanation of Fitts' law to date, the community should be aware that the supposedly resulting power model is in fact a logarithmic model reminiscent of Fitt's original formulation – as was incidentally the case for the older deterministic iterative-corrections model.

In addition, our rigorous analysis shows that the two classes of candidate mathematical descriptions of Fitts' law are *not* equivalent and this, we feel, should stimulate experimental research on the subject.

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